MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Experiment 4 *Solutions*: Forces and Torques on Magnetic Dipoles

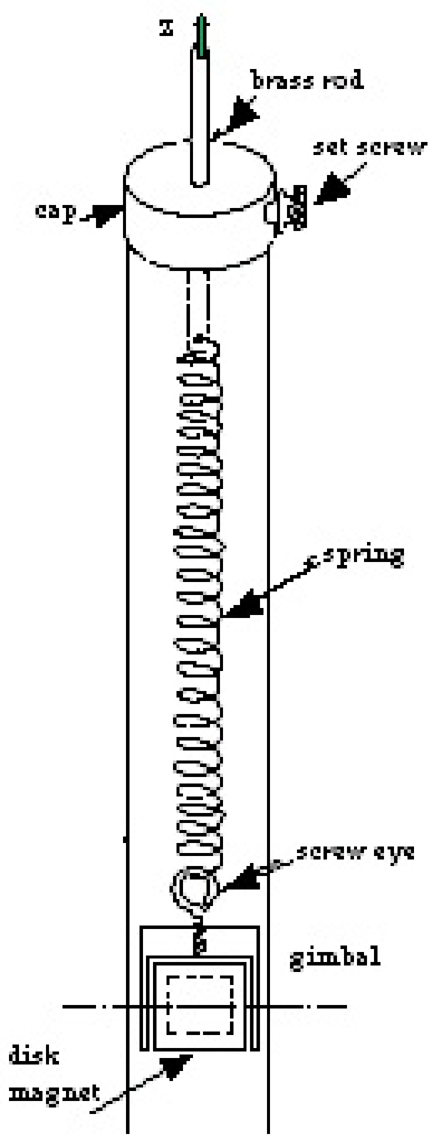
Pre-Lab Questions & Solutions

1. Force on a Dipole in the Helmholtz Apparatus

In class you calculated the magnetic field along the axis of a coil to be given by:

$$B_{axial} = \frac{N\,\mu_0\,I\,R^2}{2}\frac{1}{(z^2+R^2)^{3/2}}$$

where z is measured from the center of the coil.



In this lab we will have a disk magnet (a dipole) suspended on a spring, which we will use to observe force s on dipoles due to different magnetic field configurations.

(a) Assuming we energize only the top coil (current running counter-clockwise in the coil, creating the field quoted above), and assuming that the dipole is always well aligned with the field and on axis, what is the force on the dipole as a function of position? (HINT: In this situation $F = \mu \;\, dB/dz$)

$$F = \mu\frac{dB_{axial}}{dz} = \mu\frac{N\,\mu_0\,I\,R^2}{2}\frac{d}{dz}(z^2+R^2)^{-3/2}$$

$$= \boxed{\mu\frac{N\,\mu_0\,I\,R^2}{2}\left(\frac{-3z}{(z^2+R^2)^{5/2}}\right)}$$

That is, towards the coil center

(b) The disk magnet (together with its support) has mass *m*, the spring has spring constant *k* and the magnet has magnetic moment μ. With the current on, we lift the brass rod until the disk magnet is sitting a distance z_0 above the top of the coil. Now the current is turned off. How does the magnet move once the field is off (give both direction and distance)?

We must balance the magnetic force with the spring force:

$$F_B(z=z_0)=\mu\frac{N\mu_0 I R^2}{2}\left(\frac{-3z_0}{(z_0^2+R^2)^{5/2}}\right)=F_{\text{spring}}=k\Delta z$$

$$\Rightarrow \qquad \boxed{\Delta z=\mu\frac{N\mu_0 I R^2}{2k}\left(\frac{-3z_0}{(z_0^2+R^2)^{5/2}}\right)}$$

When the magnetic field is turned off, the force from the magnetic field will disappear and the spring will relax upwards by this distance.

(c) At what height(s) is the force on the dipole the largest?

To find this we just maximize the force function (find zeros of its derivative):

$$\frac{dF_B}{dz}=\frac{d}{dz}\left[\mu\frac{N\mu_0 I R^2}{2}\left(\frac{-3z}{(z^2+R^2)^{5/2}}\right)\right]=0$$

$$\Rightarrow \qquad 0=\frac{d}{dz}\left(z\left(z^2+R^2\right)^{-5/2}\right)=\left(z^2+R^2\right)^{-5/2}-5z^2\left(z^2+R^2\right)^{-7/2}$$

$$\Rightarrow \qquad 0=z^2+R^2-5z^2 \qquad \Rightarrow \qquad \boxed{z=\pm\frac{R}{2}}$$

(d) What is the force where the field is the largest?

Where the field is largest the force must be zero. You can either think “That’s where an aligned dipole would like to be” or “Maximum field means derivative of field is zero means no force.”

(e) Our coils have a radius R = 7 cm and N = 168 turns, and the experiment is done with I = 1 A in the coil. The spring constant k ~ 1 N/m, and μ ~ 0.5 A m^2. The mass m ~ 5 g is in the shape of a cylinder ~ 0.5 cm in diameter and ~ 1 cm long. If we place the magnet at the location where the spring is stretched the furthest when the field is on, at about what height will the magnet sit after the field is turned off?

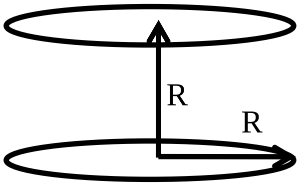
To make the spring stretch the furthest we must be at the location of the largest force, a distance z_0=R/2 above the coil (from c). From above the spring will relax by:

$$-\Delta z=\mu\frac{N\mu_0 I R^2}{2k}\left(\frac{3z_0}{(z_0^2+R^2)^{5/2}}\right)=\mu\frac{N\mu_0 I}{2kR^2}\left(\frac{3\cdot\frac{1}{2}}{(\frac{1}{4}+1)^{5/2}}\right)$$

$$\approx\left(0.5\text{ A m}^2\right)\frac{(168)\left(4\pi\times10^{-7}\text{ T m/A}\right)(1\text{ A})}{2(1\text{ N/m})(7\text{ cm})^2}\left(\frac{3\cdot 32}{2\cdot 5^{5/2}}\right)=9.2\text{ mm}$$

So the final position is a distance $3.5\text{ cm}+9.2\text{ mm}\approx 4.4\text{ cm}$ above the center of the coil

2. Motion of a Dipole in a Helmholtz Field



In Part I of this experiment we will place the disk magnet (a dipole with moment μ) at the center of the Helmholtz Apparatus (in Helmholtz mode). We will start with the disk magnet aligned along the x-axis (perpendicular to the central z-axis of the coils), and then energize the coils with a current of 1 A.

Recall that a Helmholtz coil consists of two coils of radius R and N turns each, separated by a distance R, as pictured above. The field from each coil is given at the beginning of the previous problem.

(a) Will the disk magnet experience a torque, a force or both?

It will experience a torque, trying to align it with the external field, but will not experience a force since the gradient of the field at the center of the coils is 0 (it is a field maximum).

(b) If the magnet experiences a torque:
Approximately how much time will it take for the magnet to rotate 90º, so that it is aligned with the external field? Give your answer first in terms of an approximate expression using R, N, I, and μ, and then numerically, using the values given in problem 1e above.

The torque will create an angular acceleration: $\tau = \mu B \sin(\theta) = I\alpha = I\ddot{\theta}$, which will lead to angular motion. This is a pretty ugly differential equation to solve, but we can make a stab at it in two different types of approximations. The first is to assume a constant torque, probably not the maximum torque, but maybe half of the maximum. Then we have: $\Delta\theta = \tfrac{1}{2}\alpha t^2 = \tfrac{1}{2}\frac{\tfrac{1}{2}\mu B}{I}t^2$. We can calculate the field at the center:

$$B_{\text{Helmholtz}} = 2\cdot\frac{N\,\mu_0\,I\,R^2}{2}\frac{1}{\left(\left(R/2\right)^2 + R^2\right)^{3/2}} = \frac{N\,\mu_0\,I}{R\left(\left(1/2\right)^2+1\right)^{3/2}}$$

$$\approx \frac{(168)\left(4\pi\times10^{-7}\text{ T m A}^{-1}\right)(1\text{ A})8}{(7\text{ cm})5^{3/2}} = 2.2\text{ mT} = 22\text{ Gauss}$$

The moment of inertia of a cylinder is $I = \tfrac{1}{2}mR^2 \approx \tfrac{1}{2}(5\text{ g})(0.25\text{ cm})^2 = 1.6\times10^{-8}\text{ kg m}^2$

$$t \approx \sqrt{\frac{4I\Delta\theta}{\mu B}} \approx \sqrt{\frac{4\left(1.6\times10^{-8}\text{ kg m}^2\right)(\pi/2)}{\left(0.5\text{ A m}^2\right)\left(2.2\times10^{-3}\text{ T}\right)}} \approx 9\text{ ms}$$

Another way of estimating the time is by approximating the motion as simple harmonic (which it definitely isn't because $\Delta\theta$ is so big). Then the time is a quarter of a period, which is $t = \frac{T}{4} = \frac{2\pi}{4\omega} = \frac{\pi}{2}\sqrt{\frac{I}{\mu B}} \approx \frac{\pi}{2}\sqrt{\frac{\left(1.6\times10^{-8}\text{ kg m}^2\right)}{\left(0.5\text{ A m}^2\right)\left(2.2\times10^{-3}\text{ T}\right)}} = 6\text{ ms}$

Note that this should really be a lower bound because it is for small oscillations.

Once you get larger oscillations the period starts increasing – the real period for a non-simple harmonic oscillator with amplitude θ is

$$T_{\pi/2} = T_{SHM} \cdot \left(1 + \left(\frac{1!!}{2!!}\right)^2 \sin^2\left(\frac{\theta}{4}\right) + \left(\frac{3!!}{4!!}\right)^2 \sin^4\left(\frac{\theta}{4}\right) + \ldots\right) \approx T_{SHM} \cdot 1.18$$

Meaning the time to rotate will be about 7 ms, so our approximations were both pretty good. In any case, it will be too fast for us to see the motion, instead we'll just see the end result.

(c) If the magnet experiences a force:

Approximately how much time will it take for the magnet to move to its new equilibrium position? Give your answer first in terms of an approximate expression using R, N, I, k and μ, and then numerically, using the values given in problem 1e above.

It doesn't